Oscillon formation as an initial pattern state

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Following Venkataramani and Ott [Phys. Rev. Lett. **80**, 3495 (1998)], we consider oscillons (local excitations) as spatiotemporal subcritical bifurcation phenomena. We show in a series of numerical experiments that the appearance of oscillons is highly dependent on the initial pattern state. This finding has led to a new patterning mechanism. In this Brief Report we describe the instabilities that govern the mutual interaction of oscillons/extended patterns. [S1063-651X(99)01011-9]

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A recent series of interesting experimental works on vertically vibrated granular materials [1,2] has attracted theoretical attention. One of the most interesting concerns relates to locally excited structures termed oscillons [1]. These structures are hysteretic and pinned, in contrast to localized structures in vibrated liquids [3]. A number of models have been developed to describe oscillons, using the dynamics of order parameters in continuous amplitude equations [4,5] or extensions of the Swift-Hohenberg equation [6]. Other models focused on the microscopic physics with a continuum-mechanical approximation [7] or with a nearest interaction approximation [8]. Yet, it seems that the essential concept of oscillons can be represented as a ''universal'' phenomenon of dynamical systems; this was proposed by Venkataramani and Ott [9], who suggested that oscillon formation is due to a subcritical period doubling bifurcation. In this Brief Report, we show that grid oscillon formation depends strongly on the initial amplitudes, and correlations, in the system. Furthermore, we verified that the critical ingredient in this type of oscillon formation is short range interactions between lattice points.

Venkataramani and Ott argued that generic spatially extended dynamical systems with subcritical period doubling reproduce the behavior of vibrated granular media. The amplitude at a grid point is represented as a scalar field $\xi_n(\mathbf{x})$ at position $\mathbf{x}$. The first ingredient in the difference equation is the temporal subcritical period doubling mapping M,

$$\xi_n'(\mathbf{x}) = M[\xi_n(\mathbf{x},r)] = -(r\xi+\xi^3)\exp(-\xi^2/2), \qquad (1)$$

where r is the bifurcation parameter. Next, Venkataramani and Ott introduced a spatial operator $\xi_{n+1}(\mathbf{x}) = \mathcal{L}[\xi_n'(\mathbf{x})]$, which is essential for patterning at a preferred spatial scale. In Fourier space $\mathcal{L}$ reads as

$$\overline{\xi}_{n+1}(\mathbf{k}) = f(\mathbf{k})\overline{\xi_n'}(\mathbf{k}) = \{\phi(\mathbf{k})\exp[\gamma(\mathbf{k})]\}\overline{\xi_n'}(\mathbf{k}). \qquad (2)$$

Note that M and f were not derived from physical considerations of microscopical dynamics in vibrated granular system. For an isotropic system,

$$\gamma(k) = \frac{1}{2}\left(\frac{k}{k_0}\right)^2\left[1-\frac{1}{2}\left(\frac{k}{k_0}\right)^2\right], \qquad (3)$$

$$\phi(k) = \mathrm{sgn}(k_c^2-k^2), \qquad (4)$$

where $k=|\mathbf{k}|$ and $k_c>k_0$. The essential ingredient in Eqs. (3,4) is the existence of a dominant unstable wavelength. We used similar equations without k_c and with different spectral shapes $\gamma(k)$ in the exponent and obtained similar results.

The bifurcation diagram (Fig. 1) of map M displays the stable (solid line) and the unstable (dotted line) branches of the dynamical behavior. As long as $r<r_a$, any initial condition converges to the uniform nonoscillating state (zero). For $r_a<r<r_b$, the system undergoes a subcritical period doubling. We refer to branches of unstable fixed points (dotted lines) as a threshold ξ_c, which separates the two stable states. For initial conditions in which $\xi_0(\mathbf{x})\in\Omega$, the system converges to the flat state and stays there, while for all $\xi_0(\mathbf{x})\in\Sigma$ the system converges to a period 2 state and maintains its oscillation. The existence of a separatrix (dotted line) means that the dynamics of the pattern may be strongly influenced by the initial amplitude of the pattern at each grid point. This branch is defined as a critical threshold ξ_c.

When some of the points $\xi_0(\mathbf{x})$ are in Ω while others are in Σ, the exact fate of a system will be determined by the spatial operator $f(\mathbf{k})$. This term induces the pattern formation of the oscillatory branch. One would expect to see one of three possible patterns. If all sites (grid points) are in Ω, the spatial operation can be neglected and the pattern should decay to a flat state. For the mixed state (with sites also in Σ), one can expect to observe a stable extended state, localized structures, or even a flat state.

Indeed, in numerical experiments with a variety of ran-

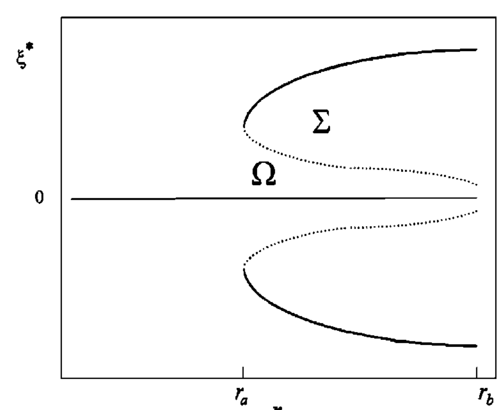

FIG. 1. Schematic bifurcation diagram of the subcritical region near period 2, based on Eq. (1).

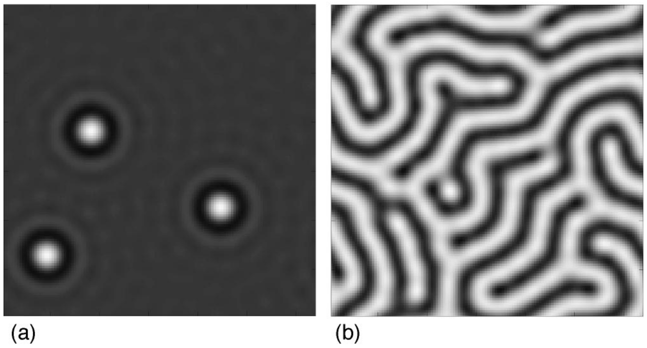

FIG. 2. Two different initial patches produced different final patterns. (a) Extended labyrinth. (b) Distribution of discrete oscillons. Patterns generated with parameters $r=0.58$, $(k_c/k_0)^2=1.7$.

dom extended initial patterns, we observed the following three stable families of patterns: spatially uniform state; extended labyrinth patterns [Fig. 2(a)]; oscillons [Fig. 2(b)]. All patterns displayed in this report were generated with parameters $r=0.58$, $(k_c/k_0)^2=1.7$. If all sites were in Ω, the pattern was flat; if the initial pattern was mixed, either oscillons or labyrinth patterns were observed. We also confirmed that the basic length scale in the pattern is $\lambda_0=2\pi/k_0$.

Our previous arguments do not tell us if there is any relationship between extended, flat, and oscillon states. Is the oscillon state stable with respect to extended or flat states? To answer this question we simulated various initial conditions. We started with a single circular excitation in Σ with diameter λ and $\xi(\mathbf{x})>\xi_c$. When $\lambda<\lambda_{min}$, where $\lambda_{min}\approx O(\lambda_0/4)$, the initial patch decayed to a uniform state due to the smearing effect of the $f(k)$ multiplication. If the scale was above λ_c, where $\lambda_c\approx\lambda_0$, a stable extended state gradually appeared. When $\lambda_{min}<\lambda<\lambda_c$, a competition between the smearing and stabilization effects gave rise to a third state of localized structures (i.e., oscillons; Fig. 3).

To investigate further the gradual appearance of the extended state, we introduced noise to both the radius and the intensity (amplitude) of our previous circular patch. We started by adding single random distributed excitation with noise η to the intensity [having a standard deviation $\sigma(\eta)$ $\sigma(\eta)=0.2\xi$], and a variation of 0.2λ to the diameter [Figs. 4(a) and 4(b)].

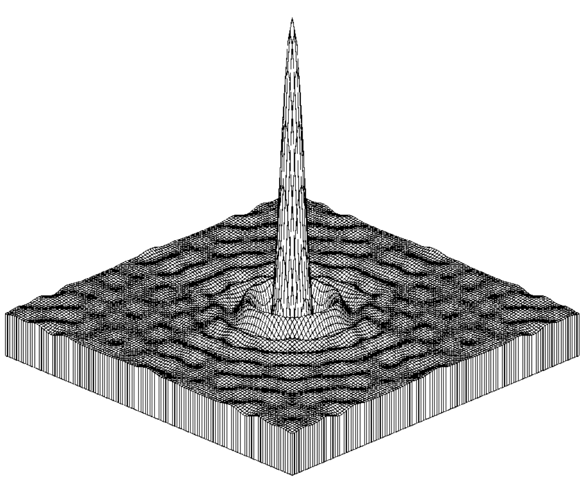

FIG. 3. Period-2 localized structure, i.e., oscillon. Parameters as in Fig. 2.

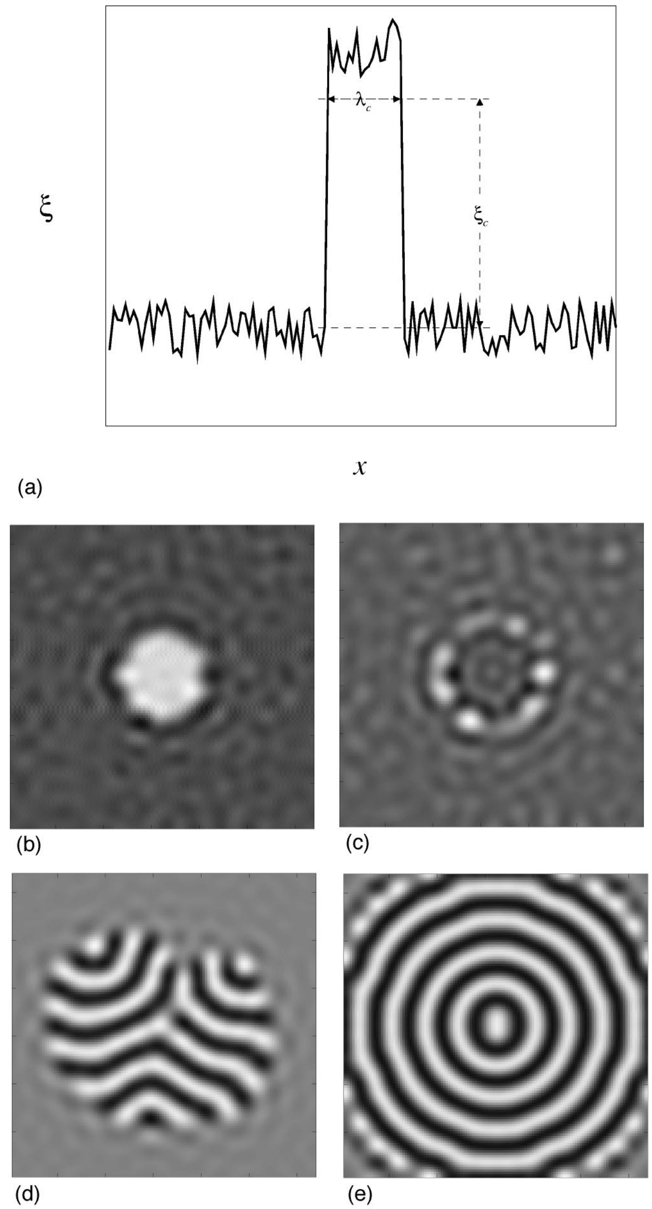

FIG. 4. Variety of patterns obtained in our numerical experiments with local initial excitation at the center of the grid. (a) Schematic diagram of initial excitation (cross section). ξ_c and λ_c are the critical scales of the amplitude and width, respectively. (b) A random core with $\lambda>\lambda_c$ and $\xi>\xi_c$. (c) Formation of oscillons inside the core regions (nucleation). (d) Expansion of the core to labyrinth/stripes. (e) Period-2 target pattern under symmetric initial two-dimensional excitation (Gaussian). Parameters as in Fig. 2.

When $\lambda>\lambda_c$, the initial excitation first formed oscillons [Fig. 4(c)] that finally collapsed to a labyrinth core patch (see also Ref. [6]). The expansion of the labyrinth pattern was followed by the nucleation phenomenon discussed in Ref. [8]. The core labyrinth pattern expands in time to the entire lattice [Fig. 4(d)]. From additional simulations we observed that though the final oscillon structure was independent of the initial form of the excitation, the labyrinth pattern depended on the initial state. If the initial excitation was without noise and had width $\lambda>\lambda_c$, the core expanded sym-

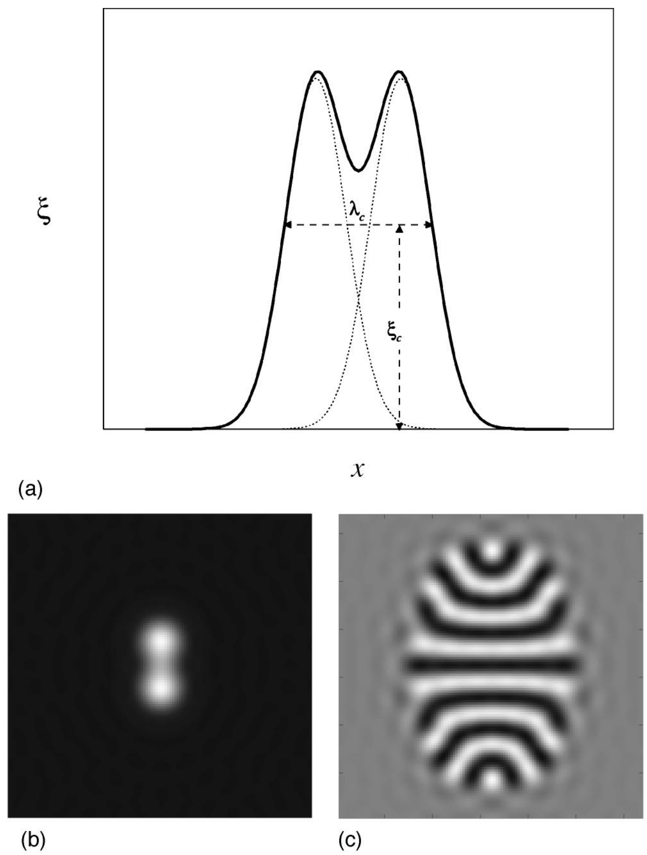

FIG. 5. (a) Schematic diagram of the initial excitation (cross section). The excitation was built from two Gaussian distributions (dotted line); the solid line displays their sum. (b) A top view of the pattern. After a few iterations the pattern began to expand. (c) After additional iterations labyrinth expansion was obtained. Parameters as in Fig. 2.

metrically to form a target pattern [Fig. 4(e)]. The outer rings formed with a transient nucleation state in which oscillons could be observed, eventually becoming smooth circular rings. For a nonsymmetrical initial structure a striped pattern appeared [Fig. 4(d)].

We also investigated the interaction between oscillons to reveal the mechanism of the aforementioned behavior. We use the following scales: $O(\xi_c)$ on excitation intensities and $O(\lambda_c)$ on excitation width for the numerical experiments generated. We introduced two Gaussian excitations with a similar phase and varied the distance between them. There was a critical distance for the instability of the oscillon states. This critical distance was limited by the demand that the amplitude of the saddle point must be below ξ_c. When the amplitude was below this value the oscillon states persisted. As the distance between the excitations decreased, the oscillon complex became wider than λ_c [Fig. 5(a)]. This instability is a simple outcome of the mechanism discussed here, and differs from the proposed mechanisms given by other authors [5–8]. Thereby, such excitations became unstable and a labyrinth/stripes of period 2 expanded through the pattern as shown in Figs. 5(b) and 5(c).

In conclusion, when the pattern has a random extended initial distribution and values within the Σ regime, there is a higher probability of obtaining a labyrinth than obtaining oscillons. This is due to the interaction along the pattern defined by the $f(\mathbf{k})$ term. The labyrinth pattern is more stable, in the sense that interaction between the oscillons and the labyrinth favors labyrinth expansion. Furthermore, a local labyrinth patch will always lead to a full labyrinth pattern. We describe a very different qualitative mechanism for oscillon formation and interaction, from that described in [5–8]. Initial excitations are an important ingredient in pattern evolution.

To summarize, in this model [9] oscillons are generic pattern structures that arise due to general considerations, without taking into account any physical microscopic dynamics. Other models [5,6,8] stress the parametric spaces of the oscillon stability while in [7] the inelasticity of intergranular interaction collisions is taken into account. From our findings, we determine that the final fate of the pattern lies in its initial excitation rather than in details of the model. Therefore, the interest in this minimal model might extend beyond granular vibrated materials.

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